

WE'VE GOT YOUR NUMBER (C Division)

DESCRIPTION: Students will use their knowledge of whole-number operations, number sense, and problem-solving ability to represent integers as expressions involving given digits and operations

EVENT PARAMETERS: Participants must provide their own pencils or pens. They will not be allowed to bring anything else to the contest, including calculators and paper. Scratch paper will be provided.

A TEAM OF UP TO: 2

APPROXIMATE TIME: 50 minutes

THE COMPETITION: Each team will be given 2 sets of problems. In each problem set, teams will be given a set of 4 digits, with some digits possibly repeated, along with the whole-number operations of addition, subtraction, multiplication, division, exponentiation, logarithm base b (where b must be formed from the digits), and factorials, plus the operation of writing single digits together to form a multi-digit number. The goal is to write as many as possible of the integers from 1 through 100 as expressions using all of the given digits and some of the given operations. In each expression, all given digits must be used exactly as many times as they appear in the set; the operations may be used as many or as few times as desired. The usual algebraic order of operations (exponentiation, multiplication/division, addition/subtraction) will take precedence, but parentheses may be used as desired to override this order. Since these are whole-number operations, decimal points are not allowed.

Symbols allowed for operations are as follows:

- (1) addition... $+$, as in $3 + 4$
- (2) subtraction... $-$, as in $4 - 3$ (Note: This is the binary operation of subtraction. Additive inverse is NOT an allowable operation.)
- (3) multiplication... \times , $*$, or parentheses, as in 3×4 or $3 * 4$ or $3(4)$. Implied multiplication is also allowable if appropriate, such as in $5 \log_3 81$. The dot (\cdot) is NOT allowed, as it is easily confused with subtraction.
- (4) division... slanted fraction bar ($/$), or a horizontal fraction bar, as in $8/4$ or $\frac{8}{4}$. The division sign (\div) is NOT allowed, as it is easily confused with addition.
- (5) exponentiation... either with superscript or the caret (\wedge), as in 4^3 or $4^{\wedge}3$. (Note: negative exponents are NOT allowed.)
- (6) factorial... $!$, as in $4!$
- (7) logarithm... the usual log with base, as in $\log_3 9$. The base must be formed as part of the use of the supplied digits; this includes logarithms base ten.

For each of the two sets of digits, each team will be given a test form, with the given digits printed at the top of each page, consisting of 100 spaces numbered 1 through 100 (four pages of 25 spaces each). Opposite each of these 100 numbers the team should write the expression that it obtains for that number. Teams will have a total of 40 minutes to prepare their papers, with warnings about the time remaining during the contest, along with a countdown from 10 seconds to zero. At that time, pencils must be put away.

EXAMPLES:

- (1) Given digits 2, 3, 4, and 9, the number 26 can be expressed as $234 / 9$, and 18 can be expressed as $3^{(4-2)} + 9$.
- (2) Given digits 2, 3, 3, and 5, the number 37 can be expressed as $3^3 + 5(2)$, and 33 can be expressed as $2^5 + 3/3$.
- (3) Given digits 1, 3, 5, and 8, the number 20 can be expressed as $5 \log_3 81$, and 87 can be expressed as $\frac{(5+1)!}{8} - 3$.

SCORING: Each of the 100 numbers in each of the 2 sets represented correctly will score 1 point, for a maximum score of 200. Each incorrect expression will result in a 1-point deduction. Incorrect expressions will include (1) numerically incorrect expressions, (2) illegible expressions, (3) expressions that fail to contain each given digit the correct number of times, (4) expressions without equal numbers of right and left parentheses, (5) expressions using illegal operation symbols, such as the dot (\cdot) for multiplication or the division sign (\div), and (6) multiple expressions for the same integer answer, regardless of whether they are correct. Blank answers will have no effect on the score. Scratch-outs of wrong expressions are allowed, provided they are clearly indicated, and will be counted the same as if the scratched out portions were blank space. Rulings regarding legibility or scratch-outs will be solely at the discretion of the event officials. Ties will be broken by comparing papers to see which has the most correct expressions for 100 (among the possible 2). If the tie is not broken, the expressions for 99 will be similarly compared, and so forth. In the extremely unlikely event that teams have exactly the same amount of correctly represented answers for all 100 numbers, the highest-numbered expressions on which the papers differ will be compared, with the advantage given to the more creative solution in the opinion of the officials. Participants still writing after time expires will be penalized 1 point for each five seconds elapsing until pencils are put away. Possession of a calculator will result in disqualification.

NOTES TO EVENT COORDINATORS:

- (1) Ample room should be given on the test paper for contestants to write expressions legibly. The format should be four pages of 25 spaces each.
- (2) It is imperative that enough grading help be assembled prior to the competition. An average grader can grade about three teams' papers per hour; use this guideline to figure how many graders will be necessary to finish grading in the allotted time period for results. "Mathematics-wise undergraduate students make excellent graders."
- (3) Graders should have calculators available to check expressions, as needed.
- (4) After the initial grading of the papers, they should be examined by a different grader to make sure that answers marked as wrong are indeed wrong, and that scoring counts and totals are correct.